Contents

1 Taylor Series Approx.

Suppose f has ∞ many derivatives near a point a. Then the taylor series is given by

$f(x) = \Sigma_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

For increment notation we can write

$f(a+h) = f(a) + f'(a)(a+h-a) + \ldots$

$= \Sigma_{n=0}^{\infty} \frac{f^{(n)}(a)}{h!}(h^n)$

Consider the approximation

$e = |f'(a) - \frac{f(a+h)-f(a)}{h}| = |f'(a) - \frac{1}{h}(f(a+h)-f(a))|$

Substituting...

$= |f'(a) - \frac{1}{h}((f(a) + f'(a)h + \frac{f''(a)}{2}h^2 + \cdots) - f(a))|$

$f(a) - f(a) = 0\ldots$ and $distributetheh$

$= |-1/2f''(a)h + \frac{1}{6}f'''(a)h^2 \cdots|$

1.1 With Remainder

We can determine for some u $f(a+h) = f(a) + f'(a)h + \frac{1}{2}f''(u)h^2$

and so the error is $e = |f'(a) - \frac{f(a+h)-f(a)}{h}| = |\frac{h}{2}f''(u)|$

- [https://openstax.org/books/calculus-volume-2/pages/6-3-taylor-and-maclaurin-seri
    - $->$ Taylor's Theorem w/ Remainder

1.2 Of Deriviatives

Again, $f'(a) \approx \frac{f(a+h)-f(a)}{h}$,

$e = |\frac{1}{2}f''(a) + \frac{1}{3!}h^2 f'''(a) + \cdots$

$R_2 = \frac{h}{2}f''(u)$

$|\frac{h}{2}f''(u)| \leq Mh^1$

$M = \frac{1}{2}|f'(u)|$

1.2.1 Another approximation

$$
\begin{aligned}
\text{err} &= |f'(a) - \tfrac{f(a)-f(a-h)}{h}| \\
&= f'(a) - \tfrac{1}{h}(f(a) - (f(a) + f'(a)(a-(a-h)) + \tfrac{1}{2}f''(a)(a-(a-h))^2 + \cdots)) \\
&= |f'(a) - \tfrac{1}{h}(f'(a) + \tfrac{1}{2}f''(a)h)|
\end{aligned}
$$